Excerpt from

COMPRESSED AIR.

Pages 113 through page 120
CHAPTER XIII

EFFECT OF LOSS OF HEAT, GENERATED DURING COMPRESSION, ON THE ULTIMATE USEFUL ENERGY RESIDING IN A GIVEN QUANTITY OF COMPRESSED AIR

117. By an accepted law of thermodynamics, work and heat are mutually convertible at the ratio of about 778 ft.-lb. of work for every B.T.U.

In Article 41a it was stated that the work expended in compressing air is all converted into heat. According to the law quoted, we should expect the compressed, and therefore heated, air to be capable of performing useful work, equal to the amount expended in compressing it. Neglecting friction in the air engine, this would actually be the case, if the compressed air could be used immediately after compression and before it has lost any of its heat.

If, on the other hand, the compressed air be allowed to cool down to the temperature which it possessed before compression, as happens in all compressed air installations, it would seem logical, by applying the same law quoted above, to reason as follows:

Since the work of compression is all converted into heat, the ability for doing useful work must have disappeared after all this heat has been abstracted.

In the following articles it will be shown:

a. That the work of compression is all converted into heat.

b. That, after all the heat of compression has been abstracted, there still remains in the compressed air a certain amount of energy for doing useful work.

c. That this is due to the energy residing in the air before compression.

a. Referring to Fig. 17, the total work of compressing adiabatically a volume $V_1$ cubic feet of free air from an absolute pressure $P_1$ to an absolute pressure $P_2$ is represented by the area $MABR$. Expressed in foot-pounds, it is equal to 144 times the numerical value of this area.

In Article 44 we found: Area $MABR = \frac{P_2V_2 - P_1V_1}{n-1}$

therefore, total work of compression $W_1 = 144 \frac{P_2V_2 - P_1V_1}{n-1}$ foot-pounds.

Let $P_1 = 14.7$ lb. absolute pressure per square inch.
$P_2 = 89.7$ lb. absolute pressure per square inch.
$= 75$ lb. gage.
$V_1 = 13.09$ cu. ft. which is the volume of 1 lb. of free air at sea level and at 60°Fahr.
$n = 1.406.$

![Fig. 17](image)
From equation (7), Article 41, deduce: \[ \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{\frac{1}{n}} \]

whence \[ V_2 = V_1 \left( \frac{P_1}{P_2} \right)^{\frac{1}{n}} = 13.09 \left( \frac{14.7}{89.7} \right)^{0.71} = 3.62 \text{ cu. ft.} \quad (3) \]

Substituting values in equation (2) we get: \[ W_1 = 144 \frac{89.7 \times 3.62 - 14.7 \times 13.09}{0.406} = 47,000 \text{ ft.-lb.} \quad (4) \]

After the air has been compressed adiabatically to an absolute pressure \( P_2 \) its absolute temperature will be according to equation (11), Article 41:

\[ T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 60 + 461 \left( \frac{89.7}{14.7} \right)^{0.29} = 880 \ \text{° absolute} \]

\[ = 419 \ \text{° Fahr.} \quad (5) \]

After compression, the original pound of air occupies a volume \( V_1 = 3.62 \text{ cu. ft.} \) and has a temperature of 419° Fahr. which is (419-60) = 359 degrees more than its initial temperature.

Now, we can imagine a volume \( V_2 \) of air weighing 1 lb. to have a temperature of 60° Fahr. If we raise the temperature of this air by \( (T_2 - T_1) = (880-561) = 359 \) degrees without changing its volume, we heat under constant volume. The specific heat \( C_v \) of air in this case is 0.168 and the amount of heat put into this pound of air, expressed in B.T.U.’s is \[ C_v (T_2 - T_1) = 0.168 \times 359 = 60.3 \text{ B.T.U.’s.} \]

Expressed in foot-pounds it is: \[ K_v (T_2 - T_1) = 131.6 \times 359 = 47,000 \text{ ft.-lb.} \quad (6) \]

A comparison of equation (6) with (4) shows that the mechanical equivalent of the heat required to raise the temperature of 1 lb. of air from an absolute temperature \( T_1 \) to an absolute temperature \( T_2 \) is identical with the mechanical energy expended in compressing adiabatically 1 lb. of atmospheric air having an absolute temperature \( T_1 \) to a pressure which raises the temperature of the air to an absolute temperature \( T_2 \). In other words, the mechanical work of compressing air adiabatically is all converted into heat energy.

b. If we now allow this volume \( V_2 = 3.62 \text{ cu. ft.} \) of compressed air, having a temperature of 419° Fahr., to cool down to initial temperature of 60° Fahr. under constant volume, its pressure will decrease to a pressure \( P_3 \), which we find from the formula: \[ P_3 = P_2 \left( \frac{T_1}{T_2} \right) = 89.7 \times \frac{521}{880} = 53.2 \text{ lb. absolute} \]

The energy residing in this volume \( V_3 = 3.62 \text{ cu. ft.} \) of air for doing useful work in expanding adiabatically down from an absolute pressure of 53.2 lb. to atmospheric pressure is represented by the area \( BCGF \) in the diagram, Fig. 18, and expressed in foot-pounds it is 144 times the numerical value of this area.

From article 110 we deduce: \[ \text{Area } BCGF = \frac{P_3 V_2 - P_1 V_1}{n-1} \]

Hence energy \[ W = 144 \frac{P_3 V_2 - P_1 V_1}{n-1} \]
Applying it to the case in hand:

- \( P_3 = 53.2 \text{ lb. absolute per sq. in.} \)
- \( V_2 = 3.62 \text{ cu. ft.} \)
- \( P_1 = 14.7 \text{ lb. per sq. in.} \)

\[
V_1 = V_2 \left( \frac{P_3}{P_1} \right)^{n} = 3.62 \left( \frac{53.2}{14.7} \right)^{0.71} = 9.02 \text{ cu. ft.} \quad \text{(From equation 13, Article 41.)}
\]

\[
n = 1.406.
\]

Hence

\[
W = 144 \frac{53.2}{0.406} \times \frac{3.62}{14.7} \times \frac{9.02}{47,000} = 21,300 \text{ ft.-lb.}
\]

Comparing this with the work of compression, we have:

Substituting values in equation (2) we get:

\[
\frac{21,300}{47,000} = 0.45 = 45 \text{ per cent.}
\]

That is, theoretically, after cooling down to initial temperature, there still remains in the compressed air energy for doing expansive work to the amount of 45 per cent. of the energy expended in compressing it.

Referring to the diagram in Fig. 17, it will be noted that part of the total work of compression represented by the area \( MABR \) is performed by the atmospheric air rushing into the cylinder behind the piston during the compression stroke and not by energy furnished by the compressor. This work is represented by the area \( MAFR \).

In practice, the air, after being compressed, is delivered into the receiver. The work of delivery is jointly performed by the compressor and by the atmospheric air. The compressor's work is represented by the area \( FBCD \) and the work of the atmosphere by the area \( RFDO \). The net work of compression and delivery done by the air compressor alone is represented by the area \( ABCD \). The compressor's share of delivery work is always available for doing useful work in the air engine because in forcing a volume of compressed air from the air-cylinder into the receiver, an equal volume of air is displaced therein, and this displacement process is extended into the pipe line and finally into the air engine, where, in making room for itself, this volume of compressed air drives the piston forward, and thus does useful work.

It may be asked: What becomes of the energy contributed by the atmospheric air toward compression and delivery which is represented by the area \( MADO \) in Fig. 17?

This energy is actually stored up in the compressed air when the latter leaves the compressor. It could do useful work if it were practicable to exhaust the air from the engines into a vacuum. But since we must exhaust against atmospheric pressure, the energy is consumed in the process of exhaustion and is therefore not available for useful work. It is not included in the formulas expressing power to be furnished by the compressor because it is furnished gratis by the atmosphere; and it is not included in the formulas expressing the useful work which a volume of compressed air can perform, because it is not available for such work.
The following example shows the effect of heat loss upon the total power stored up in a mass of air by the compressor.

**Example.** - To compress adiabatically in one stage 100 cu. ft. of free air per minute at sea level to 60 lb. gage and deliver it into the receiver, requires (theoretically) 13.40 h.p. (from column 4 Table V).

If the temperature of the free air was 60° before compression, after compression it will be 375° Fahr. (column 6 Table V) and the volume of the compressed air will be 31.44 cu. ft. (column 5 Table III)

If used immediately after compression, before having lost any heat, it could do work (theoretically) to the amount of 13.40 h.p. by expanding adiabatically down to atmospheric pressure.

But if allowed to cool, before use, to initial temperature under constant volume, the pressure will decrease to a pressure \( P_3 \) which we find from the following formula:

\[
P_3 = P_2 \frac{T_1}{T_2} = \frac{60 + 14.7}{375 + 461} = 46.6 \text{ lb. absolute}
\]

A volume of 31.44 cu. ft. of air per minute at 46.6 lb. absolute, if allowed to expand adiabatically down to atmospheric pressure could perform (theoretically) an amount of work found from equation (1) Article 111:

\[
\text{Horse-power} = \frac{144 \times 1.406 \times 46.6 \times 31.44}{33,000 \times 0.406} \left( 1 - \left( \frac{P_a}{P_2} \right)^{0.29} \right) = 6.30 \text{ h.p.}
\]

which is about 47 per cent. of the power expended in compression and delivery.

When friction and other imperfections are taken into account, this percentage decreases materially.

Adding 15 per cent. to the power of production we get 15.43 h.p.

Subtracting 15 per cent. from the available theoretical energy we get 5.35 h.p. and the comparative value shrinks to 35 per cent. This is further diminished by losses during transmission which are pointed out under Articles 93-94 and 97-105.

c. The answer to the question, why energy still remains in the compressed air after all the heat of compression has been dissipated, is that a certain capacity for work resides in the air which is due to the latter's ability to expand when the proper conditions prevail.

Such conditions could be brought about by confining a volume of atmospheric air in a cylinder under a piston and then create a partial vacuum on the other side of the piston; the atmospheric air in the cylinder would expand and push out the piston, that is, perform work. But creating a vacuum requires extra work, and is therefore not of practical application in air engines.

As a matter of fact, after all the heat generated during compression of a volume of air has been dissipated, the compressed air possesses no more energy than it did before compression, but the energy which it did possess has, by mechanical compression, been made available for doing useful work.

To do work, however, the air requires energy in the form of heat and while expanding, it consumes heat that was contained in its mass before compression. As a consequence the temperature of the expanded air falls below that of the surrounding atmosphere. The amount of heat consumed is equivalent to the amount of work performed and equal to the amount of heat that would be generated in compressing this air from the pressure at which it exhausts from the air engine to the pressure at which it enters the same.

The consumption of heat from the mass of the expanding air is manifested by the cold created in and around the cylinders of an engine using air expansively. Theoretically this is exactly the reverse of the generation of heat in the air cylinders of a compressor.
117a. Determination of the Value of “*n*” used in adiabatic compression and expansion formulas:

From equation (6), Article 117, we have:

Work of adiabatic compression of 1 lb. of free air: \[ W = K_v (T_2 - T_1) \text{ foot-pounds} \] (1)

in which \( K_v \) = specific heat of air at constant volume, expressed in foot-pounds.

\( T_2 \) = final absolute temperature of air after being compressed to an absolute pressure \( P_2 \).

\( T_1 \) = initial absolute temperature of air at an absolute pressure \( P_1 \).

In the diagram, Fig. 17, the area \( MABR \) represents the mechanical work of compressing a volume \( V \) of air from an absolute pressure \( P_1 \) to an absolute pressure \( P_2 \), the volume of compressed air being \( V_2 \).

From equation (1) Article 117: Area \( MABR = \frac{P_2V_2 - P_1V_1}{n-1} \) (2)

Let \( P_1 \) and \( P_2 \) be the absolute pressures in pounds per square foot; then the work performed, corresponding to area \( MABR \):

\[ W = \frac{P_2V_2 - P_1V_1}{n-1} \text{ foot-pounds} \] (3)

Let, furthermore, \( V_1 \) and \( V_2 \) represent volumes occupied by 1 lb. of air when under an absolute pressure of \( P_1 \) or \( P_2 \) respectively; then from equation (5) Article 20:

\[ P_1V_1 = RT_1 \]

and

\[ P_2V_2 = RT_2 \]

Substituting these values in equation (3) we have:

\[ W = \frac{RT_2 - RT_1}{n-1} = \frac{R (T_2 - T_1)}{n-1} \] (4)

From equation (7) Article 20 we have: \( R = K_p - K_v \)

Substituting in equation (4) we get:

\[ W = \frac{(K_p - K_v)(T_2 - T_1)}{n-1} \] (5)

This work is equal to the work expressed by equation (1), therefore:

\[ K_v (T_2 - T_1) = \frac{(K_p - K_v)(T_2 - T_1)}{n-1} \] (5)

or

\[ nK_v - K_v = K_p - K_v \]

whence

\[ n = \frac{K_p}{K_v} \] (6)

as first stated under Article 40.